FORMULARIO DE LA ASIGNATURA

DISEÑO DE EXPERIMENTOS Y MODELOS DE REGRESIÓN

Cátedra de Estadística ETSII – UPM

Versión 2022.02

Tema 1. Análisis de la varianza

1) Comparación de dos tratamientos:

1.a) Modelo:

$$y_{ij} = \mu_i + u_{ij}, \qquad \forall i \in [1, I], j \in [1, n_i] \qquad u_{ij} \to N(0, \sigma)$$

I : número de tratamientos n_i: número de observaciones del tratamiento i-ésimo

1.b) Comparación de medias:

$$\frac{(\bar{y}_{1\bullet} - \bar{y}_{2\bullet}) - (\mu_1 - \mu_2)}{\hat{s}_R\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \to t_{n-2} \quad \text{donde} \quad \hat{s}_R^2 = \frac{n_1 - 1}{n_1 + n_2 - 2}\hat{s}_1^2 + \frac{n_2 - 1}{n_1 + n_2 - 2}\hat{s}_2^2$$

1.c) Comparación de varianzas:

$$\frac{\hat{s}_1^2}{\sigma_1^2} \Big/ \frac{\hat{s}_2^2}{\sigma_2^2} \to F_{n_1 - 1, n_2 - 1}$$

2) Comparación de 'k' tratamientos:

2.a) Modelo: $y_{ij} = \mu_i + u_{ij}, \qquad u_{ij} \to N(0, \sigma)$

2.b) Descomposición de variabilidad:

$$VT = \sum_{i=1}^{K} \sum_{j=1}^{n_i} (y_{ij} - \overline{y}_{\bullet\bullet})^2$$

$$VE = \sum_{i=1}^{K} n_i (\overline{y}_{i\bullet} - \overline{y}_{\bullet\bullet})^2$$

$$VNE = \sum_{i=1}^{K} \sum_{j=1}^{n_i} (y_{ij} - \overline{y}_{i\bullet})^2 = \sum_{i=1}^{K} \sum_{j=1}^{n_i} {e_{ij}}^2$$

2.c) Tabla Análisis de Varianza:

Fuentes	Suma de Cuadrados	Grados de Libertad	Varianzas	F
Tratamientos	$\sum n_i (\overline{y}_{i\bullet} - \overline{y}_{\bullet\bullet})^2$	$K - 1$	$VE/(K-1)$	$\dfrac{\sum n_i (\overline{y}_{i\bullet} - \overline{y}_{\bullet\bullet})^2}{(K-1)\hat{s}_R^2}$
Residual	$\sum\sum (y_{ij} - \overline{y}_{i\bullet})^2$	$n - K$	$\hat{s}_R^2 = VNE/(n-K)$	
Total	$\sum\sum (y_{ij} - \overline{y}_{\bullet\bullet})^2$	$n - 1$		

2.d) Intervalos de confianza para medias:

$$\mu_i \in \overline{y}_{i\bullet} \pm t_{\alpha/2} \frac{\hat{s}_R}{\sqrt{n_i}}$$

2.e) Contraste dos a dos para la diferencia de medias:

$$t_{ij} = \frac{\bar{y}_{i\bullet} - \bar{y}_{j\bullet} - (\mu_i - \mu_j)}{\hat{s}_R\sqrt{\frac{1}{n_i} + \frac{1}{n_j}}} \to t_{n-K}$$

Tema 2. Diseño de experimentos

1) *Dos factores con interacción*

$$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + u_{ijk} \qquad \forall i \in [1, I],\ j \in [1, J],\ k \in [1, m]$$

$$u_{ijk} \to N(0,\sigma)\ ;\ \sum_{i=1}^{I} \alpha_i = 0\ ;\ \sum_{j=1}^{J} \beta_j = 0\ ;\ \sum_{i=1}^{I} \alpha\beta_{ij} = 0, \forall j\ ;\ \sum_{j=1}^{J} \alpha\beta_{ij} = 0, \forall i$$

I : número de niveles factor A J : número de niveles factor B m : número de replicaciones

1.a) Descomposición de variabilidad:

$$VT = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{m} (y_{ijk} - \bar{y}_{\bullet\bullet\bullet})^2$$

$$VNE = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{m} {e_{ijk}}^2$$

$$e_{ijk} = y_{ijk} - \bar{y}_{ij\bullet}$$

$$VE(A) = mJ \sum_{i=1}^{I} (\bar{y}_{i\bullet\bullet} - \bar{y}_{\bullet\bullet\bullet})^2 = mJ \sum_{i=1}^{I} (\hat{\alpha}_i)^2$$

$$VE(B) = mI \sum_{j=1}^{J} (\bar{y}_{\bullet j\bullet} - \bar{y}_{\bullet\bullet\bullet})^2 = mI \sum_{i=1}^{I} (\hat{\beta}_i)^2$$

$$VE(A \times B) = m \sum_{i=1}^{I} \sum_{j=1}^{J} (\alpha\beta_{ij})^2$$

1.b) Tabla de Análisis de Varianza:

Fuentes Variabilidad	Suma de Cuadrados	Grados de Libertad.	Varianza	F	$p -$ valor
A	$mJ\sum(\bar{y}_{i\bullet\bullet} - \bar{y}_{\bullet\bullet\bullet})^2$	$I - 1$	$\hat{s}_A^2 = VE(A)/(I-1)$	$\hat{s}_A^2 / \hat{s}_R^2$	p_A
B	$mI\sum(\bar{y}_{\bullet j\bullet} - \bar{y}_{\bullet\bullet\bullet})^2$	$J - 1$	$\hat{s}_B^2 = VE(A)/(J-1)$	$\hat{s}_B^2 / \hat{s}_R^2$	p_B
$A \times B$	$m\sum\sum(\bar{y}_{ij\bullet} - \bar{y}_{i\bullet\bullet} - \bar{y}_{\bullet j\bullet} + \bar{y}_{\bullet\bullet\bullet})^2$	$(I-1)(J-1)$	$\hat{s}_{AB}^2 = VE(AB)/(I-1)(J-1)$	$\hat{s}_{AB}^2 / \hat{s}_R^2$	p_{AB}
Residual	$\sum\sum\sum e_{ijk}^2$	$IJ(m-1)$	$\hat{s}_R^2 = VNE/(IJ(m-1))$		
Total	$\sum\sum\sum(y_{ijk} - \bar{y}_{\bullet\bullet\bullet})^2$	$n - 1$			

1.c) Comparaciones múltiples (interacción nula): factor A

$$\left(\bar{\boldsymbol{y}}_{\boldsymbol{i\bullet\bullet}} - \bar{\boldsymbol{y}}_{\boldsymbol{j\bullet\bullet}} - (\boldsymbol{\alpha_i} - \boldsymbol{\alpha_j})\right) \quad / \quad \hat{\boldsymbol{s}}_{\boldsymbol{R}}\sqrt{\boldsymbol{2/mJ}} \to \boldsymbol{t_{IJ(m-1)}}$$

1.d) Intervalos de confianza (interacción nula): factor A

$$\mu + \alpha_i \in \bar{y}_{i\bullet\bullet} \pm t_{\frac{\alpha}{2}} \cdot \hat{s}_R \,/ \sqrt{mJ}$$

1.e) Intervalos de confianza (interacción significativa):

$$\mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} \in \bar{y}_{ij\bullet} \pm t_{\alpha/2} \cdot \hat{s}_R \,/ \sqrt{m}$$

2) *Bloques aleatorizados*

$$y_{ij} = \mu + \alpha_i + \beta_j + u_{ij} \quad \forall i \in [1, I],\ j \in [1, J]; \qquad u_{ijk} \to N(0,\sigma)\ ;\ \sum_{i=1}^{I} \alpha_i = 0 \quad \sum_{j=1}^{J} \beta_j = 0$$

I : número de niveles Factor J : número de niveles Bloque

2.a) Descomposición de variabilidad:

$$VT = \sum_{i=1}^{I} \sum_{j=1}^{J} (y_{ij} - \bar{y}_{\bullet\bullet})^2$$

$$VNE = \sum_{i=1}^{I} \sum_{j=1}^{J} e_{ij}^2$$

$$VE(T) = J \sum_{i=1}^{I} (\bar{y}_{i\bullet} - \bar{y}_{\bullet\bullet})^2$$

$$VE(B) = I \sum_{j=1}^{J} (\bar{y}_{\bullet j} - \bar{y}_{\bullet\bullet})^2$$

$$e_{ij} = y_{ij} - \bar{y}_{i\bullet} - \bar{y}_{\bullet j} + \bar{y}_{\bullet\bullet}$$

2.b) Tabla de Análisis de Varianza:

Fuentes Variabilidad	Suma de Cuadrados	Grados de Libertad.	Varianza	F	$p -$ valor
Factor	$J\sum(\bar{y}_{i\bullet} - \bar{y}_{\bullet\bullet})^2$	$I - 1$	$\hat{s}_T^2 = VE(T)/(I-1)$	$\hat{s}_T^2 / \hat{s}_R^2$	p_T
Bloque	$I\sum(\bar{y}_{\bullet j} - \bar{y}_{\bullet\bullet})^2$	$J - 1$	$\hat{s}_B^2 = VE(B)/(J-1)$	$\hat{s}_B^2 / \hat{s}_R^2$	p_B
Residual	$\sum\sum e_{ij}^2$	$(I-1)(J-1)$	$\hat{s}_R^2 = VNE/(I-1)(J-1)$		
Total	$\sum\sum(y_{ij} - \bar{y}_{\bullet\bullet})^2$	n-1			

2.c) Intervalo de confianza (para los tratamientos): $\mu + \alpha_i \in \bar{y}_{i\bullet} \pm t_{\alpha/2}\, \hat{s}_R \,/\sqrt{J}$

2.d) Contraste dos a dos (para los tratamientos): $\left(\bar{y}_{i\bullet} - \bar{y}_{j\bullet} - (\alpha_i - \alpha_j)\right) \ /\ \hat{s}_R\sqrt{2/J} \ \to \ t_{(I-1)(J-1)}$

Tema 3. Modelos de Regresión

1) Regresión lineal simple (RLS)

1.a) Estimación:

$$\hat{\beta}_1 = \operatorname{cov}(x_i, y_i) / \operatorname{var}(x_i) \qquad \hat{\beta}_0 = \overline{y} - \hat{\beta}_1 \overline{x} \qquad \hat{s}_R^2 = \frac{\sum_{i=1}^n e_i^2}{n-2}$$

1.b) Distribución de estimadores:

$$\hat{\beta}_1 \to N(\beta_1, \sigma^2/(n s_x^2)) \qquad \hat{\beta}_0 \to N\left(\beta_0, \frac{\sigma^2}{n}\left(1 + \frac{\bar{x}^2}{s_x^2}\right)\right) \qquad \frac{(n-2)\hat{s}_R^2}{\sigma^2} \to \chi_{n-2}^2$$

1.c) Contrastes:

$$(\hat{\beta}_1 - \beta_1) / \left(\frac{\hat{s}_R}{\sqrt{n} s_x}\right) \to t_{n-2} \qquad (\hat{\beta}_0 - \beta_0) / \left(\frac{\hat{s}_R}{\sqrt{n}} \sqrt{1 + \frac{\bar{x}^2}{s_x^2}}\right) \to t_{n-2}$$

1.d) Descomposición de la variabilidad:

$$VE = \hat{\beta}_1^2 \, n \, s_x^2 \qquad VNE = \hat{s}_R^2 \cdot (n-2) \qquad VT = \hat{s}_y^2 \cdot (n-1)$$

2) Regresión lineal múltiple (RLM)

2.a) Estimación:

$$\widehat{\boldsymbol{\beta}} = (\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{X}^T \boldsymbol{Y} \qquad \hat{s}_R^2 = \frac{\sum_{i=1}^n e_i^2}{n-k-1}$$

2.b) Distribución de estimadores:

$$\widehat{\boldsymbol{\beta}} \to N(\boldsymbol{\beta}, \sigma^2 (\boldsymbol{X}^T \boldsymbol{X})^{-1}) \qquad \frac{(n-k-1)\hat{s}_R^2}{\sigma^2} \to \chi_{n-k-1}^2$$

2.c) Varianza estimadores para $k = 2$:

$$\operatorname{var}\left[\begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix}\right] = \begin{pmatrix} \dfrac{\sigma^2}{n s_1^2 (1 - r_{12}^2)} & \dfrac{-r_{12}\sigma^2}{n s_1 s_2 (1 - r_{12}^2)} \\ \dfrac{-r_{12}\sigma^2}{n s_1 s_2 (1 - r_{12}^2)} & \dfrac{\sigma^2}{n s_2^2 (1 - r_{12}^2)} \end{pmatrix}$$

2.d) Contrastes individuales y contraste general:

$$\text{C. Individuales: } \frac{\hat{\beta}_i - \beta_i}{\hat{s}_R \sqrt{q_{ii}}} \to t_{n-k-1} \qquad \text{C. General: } \frac{VE/k}{\hat{s}_R^2} \to F_{k,n-k-1}$$

2.e) Modelo en diferencias a la media:

$$\widehat{\boldsymbol{b}} = (\widetilde{\boldsymbol{X}}^T \widetilde{\boldsymbol{X}})^{-1} \widetilde{\boldsymbol{X}}^T \widetilde{\boldsymbol{Y}} = (\boldsymbol{S}_{XX})^{-1} (\boldsymbol{S}_{XY}) \qquad \widehat{\boldsymbol{b}} \to N\left(\boldsymbol{b}, \sigma^2 \left(\widetilde{\boldsymbol{X}}^T \widetilde{\boldsymbol{X}}\right)^{-1}\right) = N(\boldsymbol{b}, \sigma^2 (\boldsymbol{S}_{XX} \cdot n)^{-1})$$

2.f) Coeficiente de determinación (R^2) y coeficiente de determinación corregido ($\bar{R}^2$):

$$R^2 = \frac{VE}{VT} = \frac{\sum(\hat{y}_i - \bar{y})^2}{\sum(y_i - \bar{y})^2} \qquad \bar{R}^2 = 1 - \frac{\hat{s}_R^2}{\hat{s}_y^2} = 1 - \frac{VNE}{VT} \cdot \frac{n-1}{n-k-1} = 1 - (1 - R^2) \cdot \frac{n-1}{n-k-1}$$

2.g) Cálculo de predicción e intervalo de confianza:

- IC para la media: $m_h \in \hat{y}_h \pm t_{\frac{\alpha}{2}} \hat{s}_R \sqrt{v_{hh}}$
- IC para una nueva observación: $y_h \in \hat{y}_h \pm t_{\alpha/2} \hat{s}_R \sqrt{1 + v_{hh}}$

donde v_{hh} se calcula:

- RLS: $v_{hh} = \frac{1}{n}\left(1 + \frac{(x_h - \bar{x})^2}{s_x^2}\right)$
- RLM (alternativa 1): $v_{hh} = \frac{1}{n}\left(1 + \left((\boldsymbol{x}_h - \bar{\boldsymbol{x}})^T \boldsymbol{S}_{XX}^{-1} (\boldsymbol{x}_h - \bar{\boldsymbol{x}})\right)\right)$ donde $\boldsymbol{x}_h = \begin{bmatrix} x_{1,h} & x_{2,h} & \cdots & x_{k,h} \end{bmatrix}^T$
- RLM (alternativa 2): $v_{hh} = \boldsymbol{x}_h^T (\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{x}_h$ donde $\boldsymbol{x}_h = \begin{bmatrix} 1 & x_{1,h} & x_{2,h} & \cdots & x_{k,h} \end{bmatrix}^T$

4. Instrucciones Esenciales R

0) Previo

```
maquinas = read.table( 'maquinas.txt', header=T )        # lectura del archivo de texto 'maquinas.txt'
head(maquinas)          # Muestra las 6 primeras filas del 'data frame' maquinas
View(maquinas)          # Abre una ventana nueva y muestra los datos
names(maquinas)         # Proporciona los nombres de las variables del 'data.frame' maquinas
maquinas$maq = factor(maquinas$maq)   # Transforma una variable numérica a un *factor*
?head                   # Con ? delante de una función nos proporciona información de la función
#------------------------------------------------------------------------------------------------------------
# Cálculo de probabilidades
dnorm(x, 0, 1)       # Función densidad de una distribución normal N(0,1)
pnorm(q, 0, 1)     # Función distribución de una distribución normal N(0,1)
qnorm(p, 0, 1)     # Función distribución inversa de una distribución normal N(0,1)

                       F. distr. Inv.     F. distrib.        F. Densidad          Números aleatorios
Binomial               pbinom             qbinom             dbinom               rbinom
Chi-Cuadrado           pchisq             qchisq             dchisq               rchisq
Exponencial            pexp               qexp               dexp                 rexp
F                      pf                 qf                 df                   rf
Geométrica             pgeom              qgeom              dgeom                rgeom
Normal                 pnorm              qnorm              dnorm                rnorm
Poisson                ppois              qpois              dpois                rpois
T-Student              pt                 qt                 dt                   rt
#------------------------------------------------------------------------------------------------------------
Instalación del paquete DisRegETSII:

    1.  Instalar el paquete "devtools" y cargarlo:
install.packages("devtools")
library(devtools)
    2.  Instalar el paquete utilizando la funcion install_github de devtools
install_github("javiercara/DisRegETSII")
```

1) Comparación de dos tratamientos

```
t.test(rend ~ maq, data = maquinas,
       var.equal=T, conf.level = 0.95)  # comparación e intervalo de confianza de dos medias
t.test(maquinas$rend ~ maquinas$maq,
       var.equal=T, conf.level = 0.95)  # alternativa a la inst. anterior (válido tmb para var.test, aov)
var.test(rend ~ maq, data = maquinas)   # comparación e intervalo de confianza para dos varianzas
```

2) Comparación de K tratamientos (modelo con factor)

```
centeno = read.table("centeno.txt",header=TRUE) # Lee el archivo
m = aov(rend ~ sem , data = centeno)    # Análisis de la varianza (aov) de *rend* en función del factor *sem*
anova(m)                                # Muestra la tabla de análisis de la varianza del modelo *m*
model.tables(m,"means")                 # Proporciona las medias de los distintos tratamientos
tapply(centeno$rend,centeno$sem,mean)   # Otra forma pra proporcionar las medias de los distintos tratam.
tapply(centeno$rend,centeno$sem,sd)     # *tapply* es muy útil, puede calcular *sd*, *var*, *length*, etc
residuals(m)                            # los residuos del modelo (sirve para cualquier modelo)
predict(m)                              # los valores predichos para cada obs. (sirve para cualquier modelo)
ICplot(m,'sem',alpha = 0.05)            # Gráfico de los IC para las medias de cada tratamiento
pairwise.t.test(centeno$rend, centeno$sem,
                p.adjust.method = 'none') # Comparación de medias dos - a - dos
```

3) Modelo con dos factores e interacción

```
venenos = read.table("venenos.txt",header=TRUE)  # Lee el archivo
m1 = aov(tiempo ~ ant*ven ,
         data = venenos)      # Realiza el aov de *tiempo* en función de dos factores con interacción
m2 = aov(tiempo ~ ant+ven ,
         data = venenos)      # Realiza el aov de *tiempo* en función de dos factores sin interacción
anova(m1)                     # Tabla de análisis de la varianza del modelo *m1*
model.tables(m1,"means")      # Proporciona las medias por filas, columnas, tratamientos y la media global
model.tables(m1,"effects")    # Proporciona las  estimaciones de los parámetros del modelo
tapply(venenos$tiempo,venenos$ant,mean)       # medias para cada antídoto (*ant*)
tapply(venenos$tiempo,list(venenos$ant,
                           venenos$ven),mean) # Medias de las combinaciones *ant* y *ven* (tratamientos)
tapply(venenos$tiempo,list(venenos$ant,
                           venenos$ven),var)  # Se puede utilizar cualquier función, por ejemplo varianza
ICplot(m1, 'ant', alpha = 0.05) # Gráfico de los IC para las medias de los cuatro *ant*
ICplot(m1, 'ven', alpha = 0.05) # Gráfico de los IC para las medias de los tres *ven*
source("interIC.R")                     # Carga en memoria interIC.R (debe estar en la carpeta)
interIC(m1, 'ant','ven', alpha = 0.05)  # Gráfico de interacción (IC para las medias de cada tratamiento)
```

4) Diagnosis del modelo

```
plot(m1)                          # Realiza los gráficos importantes para la diagnosis
plot(as.numeric(venenos$ven),
     residuals(m1))               # Gráfico de residuos para cada veneno
plot(predict(m1),residuals(m1))   # Gráfico de residuos frente a medias de tratamientos
qqnorm(residuals(m1))             # QQ plot de los residuos para comprobar normalidad
qqline(residuals(m1))             # añade linea al QQ plot de los residuos
```

5) Regresión simple

```
cars1 = read.table("cars.txt"), header = T) # carga los datos (el archivo debe estar en la carpeta)
m0 = lm (mpg ~ horse, data = cars1)  # estima el modelo de regresión: mpg = b0 + b1 horse + u
summary(m0)                          # proporciona los resultados del modelo m0
plot(cars1$horse,cars1$mpg)          # gráfico de dispersión entre horse (x) y mpg (y)
abline (m0,col="red",wd=2)           # dibuja la recta de reg. estimada en m0 (color rojo y grosor=2)
```

6) Regresión múltiple

```
m1 = lm (mpg ~ horse + weight +
        accel, data = cars1)        # estima el modelo de regresión múltiple
m1a = lm (mpg ~ horse +
             I(horse^2) + weight +
             accel, data = cars1)   # incluye el término horse al cuadrado
m1b = lm (mpg ~ horse + weight +
             I(horse*weight) +
             accel, data = cars1)   # incluye el término   horse*weight
m1c = lm (log(mpg) ~ horse + weight +
             accel, data = cars1)   # utiliza el log de mpg como variable respuesta
```

7) Regresión múltiple con variables cualitativas

```
cars1$origin = factor( cars1$origin,
    labels = c("USA","EUR","JAP"))    # Convierte "origin" a tipo "factor" y se asignan etiquetas
m2 = lm (mpg ~ horse + weight + accel + origin,
             data = cars1)         # modelo con variable cualitativa (utiliza la 1ª como referencia)
cars1$origin  =  relevel(cars1$origin,
             ref = "EUR")          # Cambia el nivel de referencia (por defecto el primero)
m2a = lm (mpg ~ horse + weight + accel + origin,
             data = cars1)         # modelo con variable cualitativa con EUR como referencia
m2b = lm (mpg ~ weight + accel + origin + horse*origin,
             data = cars1)         # modelo con parámetros asociados a horse distintos para cada origen
 m3 = lm (mpg ~ ., data = cars1) # utiliza todas las variables en cars1 como regresores
 anova(m3)                         # análisis de la varianza del modelo m3
```

8) Diagnosis del modelo de regresión

```
plot(m0)                          # diagnosis del modelo m0
resi = residuals(m0)              # residuos para las observaciones en cars1
pred = predict(m0)                # valores predichos (ajustados) para las observaciones en cars1
plot(pred,resi)                   # Diagnosis: comprueba linealidad y homocedasticidad
qqnorm(resi)                      # Diagnosis: comprueba normalidad
qqline(resi)                      # añade recta al qqplot para comprobar normalidad
```

9) Predicción

```
xnueva = data.frame(engine=180,
        horse =100,weight=3000, accel =10,
        origin = "JAP", cylinders=4)       # coche nuevo para hacer predicción del consumo
predict(m3,xnueva,interval = "confidence") # predicción e intervalo para la media
predict(m3,xnueva,interval = "prediction") # predicción e intervalo para una nueva observación
```

10) Otras instrucciones para regresión

```
m4 = step(m3)                 # a partir de m3 selecciona el modelo utilizando STEPWISE
coefficients(m4)              # coeficientes  del  modelo
confint(m4, level=0.95)       # intervalo de confianza para los coef.
vcov(m4)                      # matriz de varianza de los parámetros estimados
out = influence(m4)           # diagnosis  sobre  datos  atípicos
```

5. Tablas

1) Distribución Normal Estándar

La tabla muestra los valores z tales que $P\ (Z\ \leq\ z)$.

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997
4.0	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998	0.99998	0.99998	0.99998
4.1	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99999	0.99999

Ejemplo: $P\ (Z\ \leq\ 1{,}96)\ =\ 0{,}97500$

2) Distribución χ^2

La tabla muestra los valores x tales que $P\ (\chi_n{}^2\ \geq\ x)\ =\ \alpha$

α

n	0.995	0.99	0.975	0.95	0.5	0.05	0.025	0.01	0.005
1	0.00004	0.0002	0.001	0.004	0.455	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	1.386	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	2.366	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	3.357	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	4.351	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	5.348	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	6.346	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	7.344	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	8.343	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	9.342	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	10.341	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	11.340	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	12.340	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	13.339	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	14.339	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	15.338	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	16.338	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	17.338	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	18.338	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	19.337	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	20.337	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	21.337	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	22.337	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	23.337	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	24.337	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	25.336	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	26.336	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	27.336	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	28.336	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	29.336	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	39.335	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	49.335	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	59.335	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	69.334	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	79.334	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	89.334	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	99.334	124.342	129.561	135.807	140.169
110	75.550	78.458	82.867	86.792	109.334	135.480	140.917	147.414	151.948
120	83.852	86.923	91.573	95.705	119.334	146.567	152.211	158.950	163.648

Ejemplo: $P\ (\chi^2_9\ \geq\ 19{,}02)\ =\ 0{,}025$

3) Distribución t-Student

La tabla muestra los valores x tales que $P\ (t_n\ \geq\ x)\ =\ \alpha$.

α

n	0.2	0.15	0.1	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	1.376	1.963	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	1.061	1.386	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.978	1.250	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.920	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.906	1.134	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.889	1.108	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.883	1.100	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.879	1.093	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.876	1.088	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.851	1.050	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
50	0.849	1.047	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
60	0.848	1.045	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
70	0.847	1.044	1.294	1.667	1.994	2.381	2.648	2.899	3.211	3.435
80	0.846	1.043	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
90	0.846	1.042	1.291	1.662	1.987	2.368	2.632	2.878	3.183	3.402
100	0.845	1.042	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
Inf	0.842	1.036	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Ejemplo: $P\ (t_9\ \geq\ 2{,}262)\ =\ 0{,}025$

4) Distribución $F(\alpha = 0,05)$

La tabla muestra los valores x tales que $P\ (F_{m,n} \geq x) = 0{,}05$.

m

n	1	2	3	4	5	6	7	8	9	10
1	161.448	199.500	215.707	224.583	230.162	233.986	236.768	238.883	240.543	241.882
2	18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371	19.385	19.396
3	10.128	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812	8.786
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772	4.735
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438	3.388	3.347
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020	2.978
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588	2.544
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538	2.494
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494	2.450
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456	2.412
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423	2.378
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348
21	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420	2.366	2.321
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342	2.297
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375	2.320	2.275
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.300	2.255
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282	2.236
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.265	2.220
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.250	2.204
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.236	2.190
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.223	2.177
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.211	2.165
40	4.085	3.232	2.839	2.606	2.449	2.336	2.249	2.180	2.124	2.077
50	4.034	3.183	2.790	2.557	2.400	2.286	2.199	2.130	2.073	2.026
60	4.001	3.150	2.758	2.525	2.368	2.254	2.167	2.097	2.040	1.993
70	3.978	3.128	2.736	2.503	2.346	2.231	2.143	2.074	2.017	1.969
80	3.960	3.111	2.719	2.486	2.329	2.214	2.126	2.056	1.999	1.951
90	3.947	3.098	2.706	2.473	2.316	2.201	2.113	2.043	1.986	1.938
100	3.936	3.087	2.696	2.463	2.305	2.191	2.103	2.032	1.975	1.927
Inf	3.841	2.996	2.605	2.372	2.214	2.099	2.010	1.938	1.880	1.831

$Ejm{:}\ P\ (F_{7,8} \geq 3{,}50) = 0{,}05$

Distribución $F(\alpha = 0,05)$ (continuación)

La tabla muestra los valores x tales que $P\ (F_{m,n} \geq x) = 0{,}05$.

m

n	12	15	20	24	30	40	60	100	120	Inf
1	243.906	245.950	248.013	249.052	250.095	251.143	252.196	253.041	253.253	254.314
2	19.413	19.429	19.446	19.454	19.462	19.471	19.479	19.486	19.487	19.496
3	8.745	8.703	8.660	8.639	8.617	8.594	8.572	8.554	8.549	8.526
4	5.912	5.858	5.803	5.774	5.746	5.717	5.688	5.664	5.658	5.628
5	4.678	4.619	4.558	4.527	4.496	4.464	4.431	4.405	4.398	4.365
6	4.000	3.938	3.874	3.841	3.808	3.774	3.740	3.712	3.705	3.669
7	3.575	3.511	3.445	3.410	3.376	3.340	3.304	3.275	3.267	3.230
8	3.284	3.218	3.150	3.115	3.079	3.043	3.005	2.975	2.967	2.928
9	3.073	3.006	2.936	2.900	2.864	2.826	2.787	2.756	2.748	2.707
10	2.913	2.845	2.774	2.737	2.700	2.661	2.621	2.588	2.580	2.538
11	2.788	2.719	2.646	2.609	2.570	2.531	2.490	2.457	2.448	2.404
12	2.687	2.617	2.544	2.505	2.466	2.426	2.384	2.350	2.341	2.296
13	2.604	2.533	2.459	2.420	2.380	2.339	2.297	2.261	2.252	2.206
14	2.534	2.463	2.388	2.349	2.308	2.266	2.223	2.187	2.178	2.131
15	2.475	2.403	2.328	2.288	2.247	2.204	2.160	2.123	2.114	2.066
16	2.425	2.352	2.276	2.235	2.194	2.151	2.106	2.068	2.059	2.010
17	2.381	2.308	2.230	2.190	2.148	2.104	2.058	2.020	2.011	1.960
18	2.342	2.269	2.191	2.150	2.107	2.063	2.017	1.978	1.968	1.917
19	2.308	2.234	2.155	2.114	2.071	2.026	1.980	1.940	1.930	1.878
20	2.278	2.203	2.124	2.082	2.039	1.994	1.946	1.907	1.896	1.843
21	2.250	2.176	2.096	2.054	2.010	1.965	1.916	1.876	1.866	1.812
22	2.226	2.151	2.071	2.028	1.984	1.938	1.889	1.849	1.838	1.783
23	2.204	2.128	2.048	2.005	1.961	1.914	1.865	1.823	1.813	1.757
24	2.183	2.108	2.027	1.984	1.939	1.892	1.842	1.800	1.790	1.733
25	2.165	2.089	2.007	1.964	1.919	1.872	1.822	1.779	1.768	1.711
26	2.148	2.072	1.990	1.946	1.901	1.853	1.803	1.760	1.749	1.691
27	2.132	2.056	1.974	1.930	1.884	1.836	1.785	1.742	1.731	1.672
28	2.118	2.041	1.959	1.915	1.869	1.820	1.769	1.725	1.714	1.654
29	2.104	2.027	1.945	1.901	1.854	1.806	1.754	1.710	1.698	1.638
30	2.092	2.015	1.932	1.887	1.841	1.792	1.740	1.695	1.683	1.622
40	2.003	1.924	1.839	1.793	1.744	1.693	1.637	1.589	1.577	1.509
50	1.952	1.871	1.784	1.737	1.687	1.634	1.576	1.525	1.511	1.438
60	1.917	1.836	1.748	1.700	1.649	1.594	1.534	1.481	1.467	1.389
70	1.893	1.812	1.722	1.674	1.622	1.566	1.505	1.450	1.435	1.353
80	1.875	1.793	1.703	1.654	1.602	1.545	1.482	1.426	1.411	1.325
90	1.861	1.779	1.688	1.639	1.586	1.528	1.465	1.407	1.391	1.302
100	1.850	1.768	1.676	1.627	1.573	1.515	1.450	1.392	1.376	1.283
Inf	1.752	1.666	1.571	1.517	1.459	1.394	1.318	1.243	1.221	1.000

5) Distribución $F(\alpha = 0,025)$

La tabla muestra los valores x tales que $P\ (F_{m,n} \geq x) = 0,025$.

m

n	1	2	3	4	5	6	7	8	9	10
1	647.789	799.500	864.163	899.583	921.848	937.111	948.217	956.656	963.285	968.627
2	38.506	39.000	39.165	39.248	39.298	39.331	39.355	39.373	39.387	39.398
3	17.443	16.044	15.439	15.101	14.885	14.735	14.624	14.540	14.473	14.419
4	12.218	10.649	9.979	9.605	9.364	9.197	9.074	8.980	8.905	8.844
5	10.007	8.434	7.764	7.388	7.146	6.978	6.853	6.757	6.681	6.619
6	8.813	7.260	6.599	6.227	5.988	5.820	5.695	5.600	5.523	5.461
7	8.073	6.542	5.890	5.523	5.285	5.119	4.995	4.899	4.823	4.761
8	7.571	6.059	5.416	5.053	4.817	4.652	4.529	4.433	4.357	4.295
9	7.209	5.715	5.078	4.718	4.484	4.320	4.197	4.102	4.026	3.964
10	6.937	5.456	4.826	4.468	4.236	4.072	3.950	3.855	3.779	3.717
11	6.724	5.256	4.630	4.275	4.044	3.881	3.759	3.664	3.588	3.526
12	6.554	5.096	4.474	4.121	3.891	3.728	3.607	3.512	3.436	3.374
13	6.414	4.965	4.347	3.996	3.767	3.604	3.483	3.388	3.312	3.250
14	6.298	4.857	4.242	3.892	3.663	3.501	3.380	3.285	3.209	3.147
15	6.200	4.765	4.153	3.804	3.576	3.415	3.293	3.199	3.123	3.060
16	6.115	4.687	4.077	3.729	3.502	3.341	3.219	3.125	3.049	2.986
17	6.042	4.619	4.011	3.665	3.438	3.277	3.156	3.061	2.985	2.922
18	5.978	4.560	3.954	3.608	3.382	3.221	3.100	3.005	2.929	2.866
19	5.922	4.508	3.903	3.559	3.333	3.172	3.051	2.956	2.880	2.817
20	5.871	4.461	3.859	3.515	3.289	3.128	3.007	2.913	2.837	2.774
21	5.827	4.420	3.819	3.475	3.250	3.090	2.969	2.874	2.798	2.735
22	5.786	4.383	3.783	3.440	3.215	3.055	2.934	2.839	2.763	2.700
23	5.750	4.349	3.750	3.408	3.183	3.023	2.902	2.808	2.731	2.668
24	5.717	4.319	3.721	3.379	3.155	2.995	2.874	2.779	2.703	2.640
25	5.686	4.291	3.694	3.353	3.129	2.969	2.848	2.753	2.677	2.613
26	5.659	4.265	3.670	3.329	3.105	2.945	2.824	2.729	2.653	2.590
27	5.633	4.242	3.647	3.307	3.083	2.923	2.802	2.707	2.631	2.568
28	5.610	4.221	3.626	3.286	3.063	2.903	2.782	2.687	2.611	2.547
29	5.588	4.201	3.607	3.267	3.044	2.884	2.763	2.669	2.592	2.529
30	5.568	4.182	3.589	3.250	3.026	2.867	2.746	2.651	2.575	2.511
40	5.424	4.051	3.463	3.126	2.904	2.744	2.624	2.529	2.452	2.388
50	5.340	3.975	3.390	3.054	2.833	2.674	2.553	2.458	2.381	2.317
60	5.286	3.925	3.343	3.008	2.786	2.627	2.507	2.412	2.334	2.270
70	5.247	3.890	3.309	2.975	2.754	2.595	2.474	2.379	2.302	2.237
80	5.218	3.864	3.284	2.950	2.730	2.571	2.450	2.355	2.277	2.213
90	5.196	3.844	3.265	2.932	2.711	2.552	2.432	2.336	2.259	2.194
100	5.179	3.828	3.250	2.917	2.696	2.537	2.417	2.321	2.244	2.179
Inf	5.024	3.689	3.116	2.786	2.567	2.408	2.288	2.192	2.114	2.048

$Ejm: P\ (F_{7,8} \geq 4{,}53) = 0{,}025$

Distribución F(α = 0,025) (continuación)

La tabla muestra los valores x tales que $P\ (F_{m,n}\ \geq\ x)\ =\ 0{,}025$

m

n	12	15	20	24	30	40	60	100	120	Inf
1	976.708	984.867	993.103	997.249	1001.414	1005.598	1009.800	1013.175	1014.020	1018.258
2	39.415	39.431	39.448	39.456	39.465	39.473	39.481	39.488	39.490	39.498
3	14.337	14.253	14.167	14.124	14.081	14.037	13.992	13.956	13.947	13.902
4	8.751	8.657	8.560	8.511	8.461	8.411	8.360	8.319	8.309	8.257
5	6.525	6.428	6.329	6.278	6.227	6.175	6.123	6.080	6.069	6.015
6	5.366	5.269	5.168	5.117	5.065	5.012	4.959	4.915	4.904	4.849
7	4.666	4.568	4.467	4.415	4.362	4.309	4.254	4.210	4.199	4.142
8	4.200	4.101	3.999	3.947	3.894	3.840	3.784	3.739	3.728	3.670
9	3.868	3.769	3.667	3.614	3.560	3.505	3.449	3.403	3.392	3.333
10	3.621	3.522	3.419	3.365	3.311	3.255	3.198	3.152	3.140	3.080
11	3.430	3.330	3.226	3.173	3.118	3.061	3.004	2.956	2.944	2.883
12	3.277	3.177	3.073	3.019	2.963	2.906	2.848	2.800	2.787	2.725
13	3.153	3.053	2.948	2.893	2.837	2.780	2.720	2.671	2.659	2.595
14	3.050	2.949	2.844	2.789	2.732	2.674	2.614	2.565	2.552	2.487
15	2.963	2.862	2.756	2.701	2.644	2.585	2.524	2.474	2.461	2.395
16	2.889	2.788	2.681	2.625	2.568	2.509	2.447	2.396	2.383	2.316
17	2.825	2.723	2.616	2.560	2.502	2.442	2.380	2.329	2.315	2.247
18	2.769	2.667	2.559	2.503	2.445	2.384	2.321	2.269	2.256	2.187
19	2.720	2.617	2.509	2.452	2.394	2.333	2.270	2.217	2.203	2.133
20	2.676	2.573	2.464	2.408	2.349	2.287	2.223	2.170	2.156	2.085
21	2.637	2.534	2.425	2.368	2.308	2.246	2.182	2.128	2.114	2.042
22	2.602	2.498	2.389	2.331	2.272	2.210	2.145	2.090	2.076	2.003
23	2.570	2.466	2.357	2.299	2.239	2.176	2.111	2.056	2.041	1.968
24	2.541	2.437	2.327	2.269	2.209	2.146	2.080	2.024	2.010	1.935
25	2.515	2.411	2.300	2.242	2.182	2.118	2.052	1.996	1.981	1.906
26	2.491	2.387	2.276	2.217	2.157	2.093	2.026	1.969	1.954	1.878
27	2.469	2.364	2.253	2.195	2.133	2.069	2.002	1.945	1.930	1.853
28	2.448	2.344	2.232	2.174	2.112	2.048	1.980	1.922	1.907	1.829
29	2.430	2.325	2.213	2.154	2.092	2.028	1.959	1.901	1.886	1.807
30	2.412	2.307	2.195	2.136	2.074	2.009	1.940	1.882	1.866	1.787
40	2.288	2.182	2.068	2.007	1.943	1.875	1.803	1.741	1.724	1.637
50	2.216	2.109	1.993	1.931	1.866	1.796	1.721	1.656	1.639	1.545
60	2.169	2.061	1.944	1.882	1.815	1.744	1.667	1.599	1.581	1.482
70	2.136	2.028	1.910	1.847	1.779	1.707	1.628	1.558	1.539	1.436
80	2.111	2.003	1.884	1.820	1.752	1.679	1.599	1.527	1.508	1.400
90	2.092	1.983	1.864	1.800	1.731	1.657	1.576	1.503	1.483	1.371
100	2.077	1.968	1.849	1.784	1.715	1.640	1.558	1.483	1.463	1.347
Inf	1.945	1.833	1.708	1.640	1.566	1.484	1.388	1.296	1.268	1.000

6) Distribución $F(\alpha = 0,01)$

La tabla muestra los valores x tales que $P\ (F_{m,n} \geq x) = 0{,}01$

m

n	1	2	3	4	5	6	7	8	9	10
1	4052.181	4999.500	5403.352	5624.583	5763.650	5858.986	5928.356	5981.070	6022.473	6055.847
2	98.503	99.000	99.166	99.249	99.299	99.333	99.356	99.374	99.388	99.399
3	34.116	30.817	29.457	28.710	28.237	27.911	27.672	27.489	27.345	27.229
4	21.198	18.000	16.694	15.977	15.522	15.207	14.976	14.799	14.659	14.546
5	16.258	13.274	12.060	11.392	10.967	10.672	10.456	10.289	10.158	10.051
6	13.745	10.925	9.780	9.148	8.746	8.466	8.260	8.102	7.976	7.874
7	12.246	9.547	8.451	7.847	7.460	7.191	6.993	6.840	6.719	6.620
8	11.259	8.649	7.591	7.006	6.632	6.371	6.178	6.029	5.911	5.814
9	10.561	8.022	6.992	6.422	6.057	5.802	5.613	5.467	5.351	5.257
10	10.044	7.559	6.552	5.994	5.636	5.386	5.200	5.057	4.942	4.849
11	9.646	7.206	6.217	5.668	5.316	5.069	4.886	4.744	4.632	4.539
12	9.330	6.927	5.953	5.412	5.064	4.821	4.640	4.499	4.388	4.296
13	9.074	6.701	5.739	5.205	4.862	4.620	4.441	4.302	4.191	4.100
14	8.862	6.515	5.564	5.035	4.695	4.456	4.278	4.140	4.030	3.939
15	8.683	6.359	5.417	4.893	4.556	4.318	4.142	4.004	3.895	3.805
16	8.531	6.226	5.292	4.773	4.437	4.202	4.026	3.890	3.780	3.691
17	8.400	6.112	5.185	4.669	4.336	4.102	3.927	3.791	3.682	3.593
18	8.285	6.013	5.092	4.579	4.248	4.015	3.841	3.705	3.597	3.508
19	8.185	5.926	5.010	4.500	4.171	3.939	3.765	3.631	3.523	3.434
20	8.096	5.849	4.938	4.431	4.103	3.871	3.699	3.564	3.457	3.368
21	8.017	5.780	4.874	4.369	4.042	3.812	3.640	3.506	3.398	3.310
22	7.945	5.719	4.817	4.313	3.988	3.758	3.587	3.453	3.346	3.258
23	7.881	5.664	4.765	4.264	3.939	3.710	3.539	3.406	3.299	3.211
24	7.823	5.614	4.718	4.218	3.895	3.667	3.496	3.363	3.256	3.168
25	7.770	5.568	4.675	4.177	3.855	3.627	3.457	3.324	3.217	3.129
26	7.721	5.526	4.637	4.140	3.818	3.591	3.421	3.288	3.182	3.094
27	7.677	5.488	4.601	4.106	3.785	3.558	3.388	3.256	3.149	3.062
28	7.636	5.453	4.568	4.074	3.754	3.528	3.358	3.226	3.120	3.032
29	7.598	5.420	4.538	4.045	3.725	3.499	3.330	3.198	3.092	3.005
30	7.562	5.390	4.510	4.018	3.699	3.473	3.304	3.173	3.067	2.979
40	7.314	5.179	4.313	3.828	3.514	3.291	3.124	2.993	2.888	2.801
50	7.171	5.057	4.199	3.720	3.408	3.186	3.020	2.890	2.785	2.698
60	7.077	4.977	4.126	3.649	3.339	3.119	2.953	2.823	2.718	2.632
70	7.011	4.922	4.074	3.600	3.291	3.071	2.906	2.777	2.672	2.585
80	6.963	4.881	4.036	3.563	3.255	3.036	2.871	2.742	2.637	2.551
90	6.925	4.849	4.007	3.535	3.228	3.009	2.845	2.715	2.611	2.524
100	6.895	4.824	3.984	3.513	3.206	2.988	2.823	2.694	2.590	2.503
Inf	6.635	4.605	3.782	3.319	3.017	2.802	2.639	2.511	2.407	2.321

Ejemplo: $P\ (F_{7,8} \geq 6{,}18) = 0{,}01$

Distribución F($\alpha = 0{,}01$) (continuación)

La tabla muestra los valores x tales que $P\left(F_{m,n} \geq x\right) = 0{,}01$

m

n	12	15	20	24	30	40	60	100	120	Inf
1	6106.321	6157.285	6208.730	6234.631	6260.649	6286.782	6313.030	6334.110	6339.391	6365.864
2	99.416	99.433	99.449	99.458	99.466	99.474	99.482	99.489	99.491	99.499
3	27.052	26.872	26.690	26.598	26.505	26.411	26.316	26.240	26.221	26.125
4	14.374	14.198	14.020	13.929	13.838	13.745	13.652	13.577	13.558	13.463
5	9.888	9.722	9.553	9.466	9.379	9.291	9.202	9.130	9.112	9.020
6	7.718	7.559	7.396	7.313	7.229	7.143	7.057	6.987	6.969	6.880
7	6.469	6.314	6.155	6.074	5.992	5.908	5.824	5.755	5.737	5.650
8	5.667	5.515	5.359	5.279	5.198	5.116	5.032	4.963	4.946	4.859
9	5.111	4.962	4.808	4.729	4.649	4.567	4.483	4.415	4.398	4.311
10	4.706	4.558	4.405	4.327	4.247	4.165	4.082	4.014	3.996	3.909
11	4.397	4.251	4.099	4.021	3.941	3.860	3.776	3.708	3.690	3.602
12	4.155	4.010	3.858	3.780	3.701	3.619	3.535	3.467	3.449	3.361
13	3.960	3.815	3.665	3.587	3.507	3.425	3.341	3.272	3.255	3.165
14	3.800	3.656	3.505	3.427	3.348	3.266	3.181	3.112	3.094	3.004
15	3.666	3.522	3.372	3.294	3.214	3.132	3.047	2.977	2.959	2.868
16	3.553	3.409	3.259	3.181	3.101	3.018	2.933	2.863	2.845	2.753
17	3.455	3.312	3.162	3.084	3.003	2.920	2.835	2.764	2.746	2.653
18	3.371	3.227	3.077	2.999	2.919	2.835	2.749	2.678	2.660	2.566
19	3.297	3.153	3.003	2.925	2.844	2.761	2.674	2.602	2.584	2.489
20	3.231	3.088	2.938	2.859	2.778	2.695	2.608	2.535	2.517	2.421
21	3.173	3.030	2.880	2.801	2.720	2.636	2.548	2.475	2.457	2.360
22	3.121	2.978	2.827	2.749	2.667	2.583	2.495	2.422	2.403	2.305
23	3.074	2.931	2.781	2.702	2.620	2.535	2.447	2.373	2.354	2.256
24	3.032	2.889	2.738	2.659	2.577	2.492	2.403	2.329	2.310	2.211
25	2.993	2.850	2.699	2.620	2.538	2.453	2.364	2.289	2.270	2.169
26	2.958	2.815	2.664	2.585	2.503	2.417	2.327	2.252	2.233	2.131
27	2.926	2.783	2.632	2.552	2.470	2.384	2.294	2.218	2.198	2.097
28	2.896	2.753	2.602	2.522	2.440	2.354	2.263	2.187	2.167	2.064
29	2.868	2.726	2.574	2.495	2.412	2.325	2.234	2.158	2.138	2.034
30	2.843	2.700	2.549	2.469	2.386	2.299	2.208	2.131	2.111	2.006
40	2.665	2.522	2.369	2.288	2.203	2.114	2.019	1.938	1.917	1.805
50	2.562	2.419	2.265	2.183	2.098	2.007	1.909	1.825	1.803	1.683
60	2.496	2.352	2.198	2.115	2.028	1.936	1.836	1.749	1.726	1.601
70	2.450	2.306	2.150	2.067	1.980	1.886	1.785	1.695	1.672	1.540
80	2.415	2.271	2.115	2.032	1.944	1.849	1.746	1.655	1.630	1.494
90	2.389	2.244	2.088	2.004	1.916	1.820	1.716	1.623	1.598	1.457
100	2.368	2.223	2.067	1.983	1.893	1.797	1.692	1.598	1.572	1.427
Inf	2.185	2.039	1.878	1.791	1.696	1.592	1.473	1.358	1.325	1.000