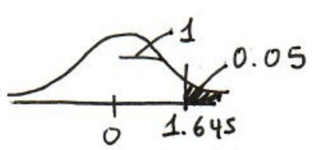
Estadística PEC-2 20-Diciembre-2018

SOLUCIÓN AL EXAMEN

(1.) $H_0: \lambda = 6$
$H_1\ \lambda \geq 6$

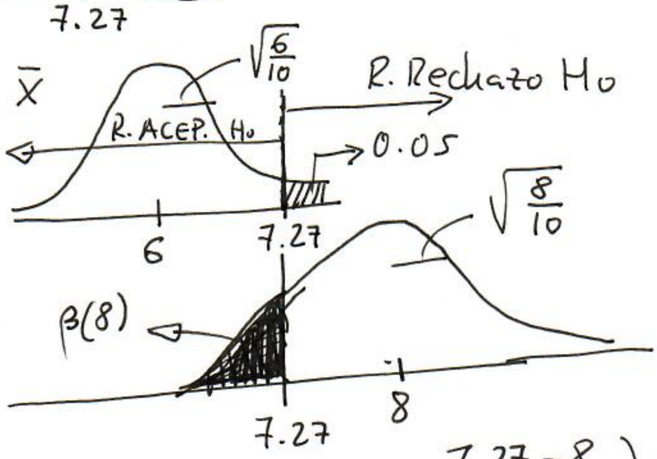
$\bar{X} \rightsquigarrow N\left(\lambda, \sqrt{\frac{\lambda}{n}}\right)$



$$\bar{x} \leq \underbrace{6 + 1.645\sqrt{\frac{6}{10}}}_{7.27} \Rightarrow \text{Acepto } H_0$$

Ho cierto
$\lambda = 6$

Ho falso
$\lambda = 8$

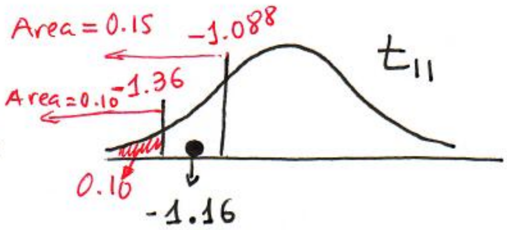


$$\beta(8) = P(\bar{X} \leq 7.27 \mid \lambda = 8) = P\left(Z \leq \frac{7.27 - 8}{\sqrt{\frac{8}{10}}}\right) = P(Z \leq -0.82) = 0.206$$

SOLUCIÓN = 0.21

(2) $H_0: \mu = 46$
$H_1: \mu < 46$

$$t_0 = \frac{42 - 46}{11.9/\sqrt{12}} = -1.16$$



$P(t_{11} \leq -1.16) = 0.134$

En las tablas se observa que

$P(t_{11} \leq -1.088) = 0.15$

$P(t_{11} \leq -1.36) = 0.10$

$$0.10 \leq \text{p-Valor} \leq 0.15$$

Solución: $0.10 \leq$ p-Valor < 0.15

(3) $f_X(x) = \frac{4}{\theta^4} x^3$, $0 \leq x \leq \theta$ $\quad E[X] = \int_0^\theta \frac{4}{\theta^4} x^4 dx = \frac{4}{5}\theta$

$$\frac{4}{5}\hat{\theta} = \bar{x} \Rightarrow \boxed{\hat{\theta} = \frac{5}{4}\bar{x}}$$

$$E[\hat{\theta}] = \frac{5}{4} E[\bar{X}] = \frac{5}{4} \times \frac{4}{5}\theta = \theta \Longrightarrow \text{Sesgo}(\hat{\theta}) = 0$$

$$\text{Var}(\hat{\theta}) = \frac{25}{16}\text{Var}(\bar{x}) = \frac{25}{16} \cdot \frac{\text{Var}(X)}{n} \quad \text{y}$$

$$\text{Var}(X) = E[X^2] - E[X]^2$$

$$E[X^2] = \int_0^\theta \frac{4x^5}{\theta^4} = \frac{4}{6}\theta^2$$

$$\text{Var}(X) = \frac{2}{3}\theta^2 - \frac{16}{25}\theta^2 = \frac{2\theta^2}{75} \Rightarrow \boxed{\text{Var}(\hat{\theta}) = \frac{25}{16} \times \frac{2\theta^2}{75n} = \frac{\theta^2}{24n}}$$

4. Distribución de Poisson

$$\hat{\alpha} = \bar{x} = \frac{0\times 5 + 1\times 14 + 2\times 20 + \cdots + 7\times 3}{104} = \frac{313}{104} = 3$$

$$\alpha \in \hat{\alpha} \pm 1.96\sqrt{\frac{\hat{\alpha}}{104}} \Rightarrow \boxed{2.67 \leq \alpha \leq 3.33}$$

5.

$$\ell(\alpha) = \alpha^n t_1 \cdots t_n e^{-\frac{\alpha}{2}\sum t_i^2}$$

$$L(\alpha) = n \log \alpha - \frac{\alpha}{2}\sum t_i^2 + k$$

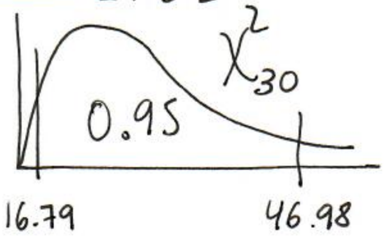
$$\frac{dL}{d\alpha} = \frac{n}{\alpha} - \frac{\sum t_i^2}{2} \Longrightarrow \frac{n}{\hat{\alpha}} - \frac{\sum t_i^2}{2} = 0$$

$$\boxed{\hat{\alpha} = \frac{2n}{\sum t_i^2} = \frac{100}{510.58} = 0.1959}$$

6.

$$\hat{S}_R^2 = \frac{10\times 33.9 + 20\times 138.9}{30} = 103.9$$

$$\frac{(n-2)\hat{S}_R^2}{\sigma^2} \sim \chi^2_{30}$$



$$16.8 \leq \frac{(n-2)\hat{S}_R^2}{\sigma^2} \leq 46.98$$

$$\boxed{66.4 \leq \sigma^2 \leq 185.6}$$

7.

$$\hat{\mu} = \frac{\sum X_i}{n-1} \qquad E[\hat{\mu}] = \frac{n}{n-1}\mu \qquad \text{sesgo}(\hat{\mu}) = \frac{\mu}{n-1}$$

$$\text{Var}(\hat{\mu}) = \frac{\sum \text{Var}(X_i)}{(n-1)^2} = \frac{n}{(n-1)^2}\sigma^2$$

$$\text{ECM}(\hat{\mu}) = \frac{\mu^2}{(n-1)^2} + \frac{n\sigma^2}{(n-1)^2} = \boxed{\frac{\mu^2 + n\sigma^2}{(n-1)^2}}$$

8.

$$\mu \in \bar{x} \pm t_{\alpha/2} \frac{\hat{S}}{\sqrt{n}} \qquad t_{\alpha/2}^{19 g.l} = 1.729$$

$$\mu \in 7.9 \pm 1.73 \frac{2.8}{\sqrt{20}} \longrightarrow \boxed{6.82 \leq \mu \leq 8.98}$$

9. Si p es la proporción de varones. La probabilidad de que al elegir a azar dos estudiantes, ambos sean varones es p^2, ambas mujeres $(1-p)^2$ y uno de cada género $2p(1-p)$. De manera que la función de verosimilitud es:

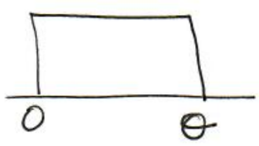
$$\ell(p) = p^{2a} (2p(1-p))^b (1-p)^{2c}$$

$$L(p) = 2a \log p + b \log 2 + b \log p + b \log(1-p) + 2c \log(1-p)$$

$$\frac{dL}{dp} = \frac{2a}{p} + \frac{b}{p} - \frac{b}{1-p} - \frac{2c}{1-p} \Rightarrow \frac{2a+b}{\hat{p}} - \frac{2c+b}{1-\hat{p}} = 0$$

$$\boxed{\hat{p} = \frac{2a+b}{2a+2b+2c}} = \frac{\text{nº total de chicos}}{\text{nº total de estudiantes}}$$

10.



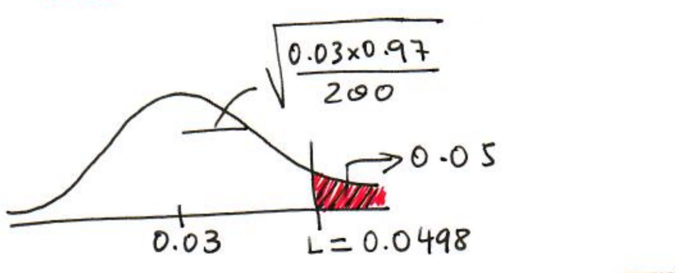
$$E[X] = \frac{\theta}{2} \Rightarrow \hat{\theta} = 2\bar{x}$$

$$E[2\bar{X}] = 2E[\bar{X}] = 2E[X] = 2\frac{\theta}{2} = \theta$$

$$Var(\hat{\theta}) = 4 Var(\bar{X}) = 4 \cdot \frac{Var(X)}{n} = \frac{\theta^2}{3n} \qquad \text{pues } Var(X) = \frac{\theta^2}{12}$$

Sol: $E[\hat{\theta}] = \theta \quad Var(\hat{\theta}) = \frac{\theta^2}{3n}$

11. $\hat{p} \to N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$ Ho es cierto

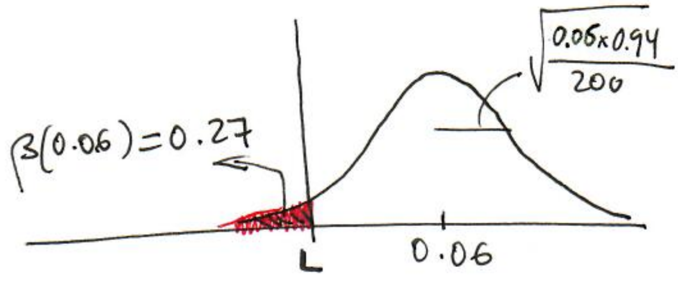


Ho cierto: $L = 0.03 + 1.645 \times \sqrt{\frac{0.03 \times 0.97}{200}}$
$= 0.0498$

$$\beta(0.06) = P\left(\hat{p} \leq 0.0498 \mid \hat{p} \to N\left(0.06, \sqrt{\frac{0.06 \times 0.94}{200}}\right)\right)$$

$$\beta(0.06) = P\left(Z \leq \frac{0.0498 - 0.06}{\sqrt{\frac{0.06 \times 0.94}{200}}}\right) = P(Z \leq -0.60)$$

$$= \boxed{0.27}$$



12. $$Cov(u_i,v_i)=\frac{\sum (u_i-\bar u)(v_i-\bar v)}{n}=\frac{\sum 0.5(x_i-\bar x)\,0.5(y_i-\bar y)}{n}$$
$$=0.5^2\,cov(x_i,y_i)$$

$S_u=0.5\,S_x$
$S_v=0.5\,S_y$

$$r_{uv}=\frac{Cov(u_i,v_i)}{S_u S_v}=\frac{0.5^2\,cov(x_i,y_i)}{0.5S_x\cdot 0.5S_y}$$
$$=r_{xy}$$

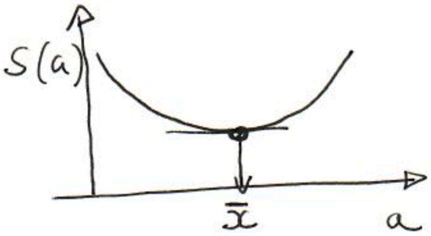
$$\boxed{r_{uv}=0.8}$$

13. $S(a)=\sum(x_i-a)^2$

$\frac{dS}{da}=-2\sum(x_i-a)=0$ $\quad\sum x_i-na=0$

$a=\frac{\sum x_i}{n}=\bar x$

$\frac{d^2S}{da^2}=2n>0 \Rightarrow$ es Minimo

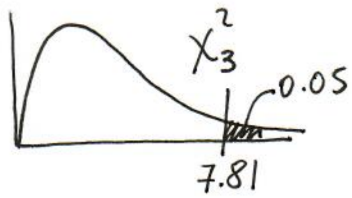


Solución
La media

15.

	O_i	
61	315	312.75
62	108	104.25
63	101	104.25
64	32	34.75
	556	556

$E_i=556\times p_i$ $\quad\chi^2_0=\frac{\sum(O_i-E_i)^2}{E_i}=0.47\to\chi^2_{4-1=3}$

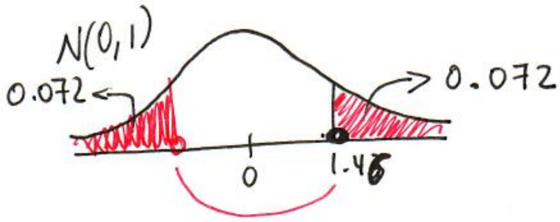


Región aceptación [0,7.81]
se acepta la teoría,
pues $0.47\le 7.81$

14. $\mu\in\bar x\pm 2.57\frac{\sigma}{\sqrt n}$ $\quad 2\times2.57\times\frac{8.35}{\sqrt n}=6$

$$n=\frac{2^2\times 2.57^2\times 8.35^2}{6^2}=51.$$

16. $z_0=\frac{208-20}{3/\sqrt{30}}=1.46$



$$\boxed{p\text{-valor}=0.144}$$